

# Robust techniques of automatic control for mobile robotic systems

Juan Luis Rosendo

Grupo de Control Aplicado (GCA)

Instituto LEICI (UNLP - CONICET)

Facultad de Ingeniería, Universidad Nacional de La Plata

ENSTA Bretagne



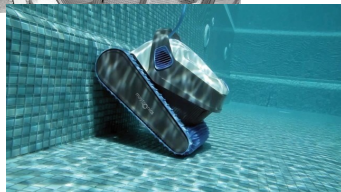
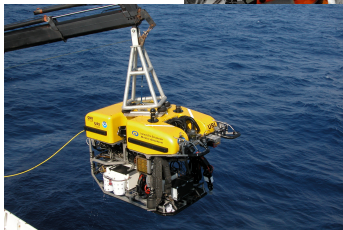
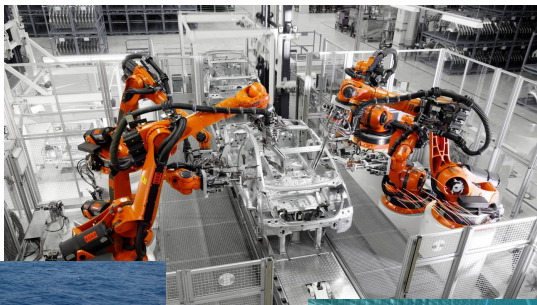
## Plan

- 1 Introduction.
- 2 AUV Ciscreea modeling.
- 3 Input constraint compensating algorithm.
- 4 Output constraint compensating algorithm.
- 5 Control design under structural constraints.
- 6 Control design under dynamic constraints.
- 7 Concluding remarks.

# Plan

- 1 Introduction.
- 2 AUV Ciscrea modeling.
- 3 Input constraint compensating algorithm.
- 4 Output constraint compensating algorithm.
- 5 Control design under structural constraints.
- 6 Control design under dynamic constraints.
- 7 Concluding remarks.

# Robots.

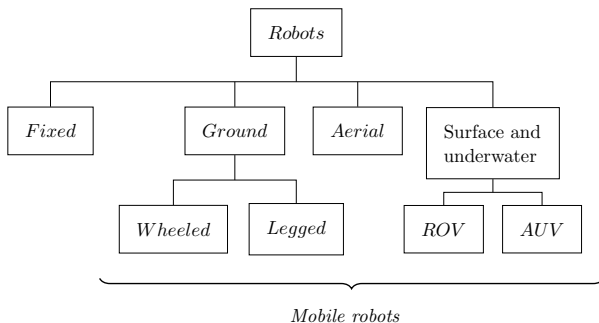




## Types of Robots.

From the mechanical point of view:

- Fixed robots.
- Mobile robots.



## Fields of application.

Considering their potential use and the degree of technologies development:

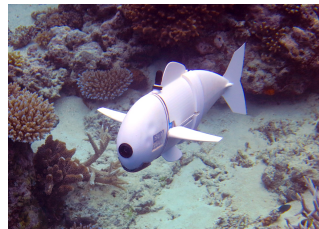
- Industrial robotics.



## Fields of application.

Considering their potential use and the degree of development of the technologies:

- Industrial robotics.
- Advanced robotics.



## About this work.

This thesis focuses on:

- Mobile robot control.
- Advanced robot applications.

In particular it will be focused on the control objectives of:

- Tracking task.
- Obstacle avoidance task.

## About this work

The control of these systems is strongly affected by:

- Constraint effects.
- Non idealities.
- External perturbations.

## About this work

The control of these systems is strongly affected by:

- Constraint effects.
- Non idealities.
- External perturbations.

To deal with these problems the following is proposed:

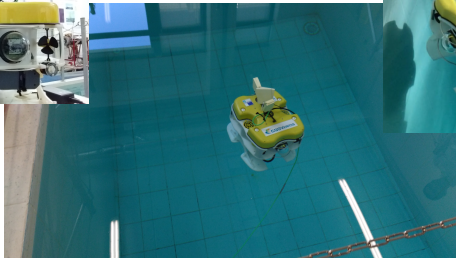
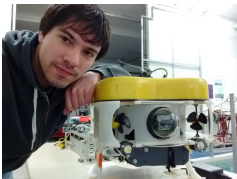
- 1 External loops to the main control of these systems in order to reduce the input/output constraint effects.
- 2 A robust controller tuning technique considering structural constraints.
- 3 Controllers synthesis and analysis considering non-idealities and dynamic constraints.

# Plan

- 1 Introduction.
- 2 AUV Ciscree modeling.
- 3 Input constraint compensating algorithm.
- 4 Output constraint compensating algorithm.
- 5 Control design under structural constraints.
- 6 Control design under dynamic constraints.
- 7 Concluding remarks.

## AUV Ciscrea

Size	0.525m (L) 0.406m (W) 0.395m (H)
Weight in air	15.56kg (without payload and floats)
Degrees of Freedom	Surge, Sway, Heave and Yaw
Propulsion	2 vertical and 4 horizontal propellers
Speed	2 knots (Surge) and 1 knot (Sway, Heave)
Depth Rating	50m
On-board Battery	2-4 hours





## AUV Ciscrea model

Rigid-body dynamic:

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau_{env} + \tau_{hydro} + \tau_{pro} \quad (1)$$

Hydrodynamic formulations:

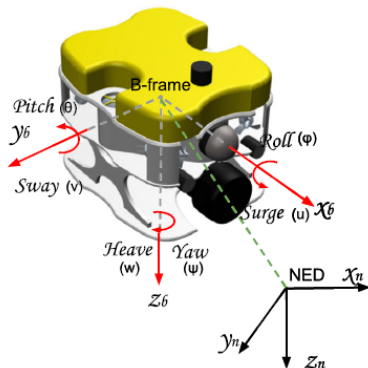
$$\tau_{hydro} = -M_A\dot{\nu} - C_A(\nu)\nu - D(|\nu|)\nu - g(\eta) \quad (2)$$

Damping:

$$D(|\nu|) = D_L + D_N|\nu| \quad (3)$$

Parameter	Description
$M_{RB}$	AUV rigid-body mass and inertia matrix
$M_A$	Added mass matrix
$C_{RB}$	Rigid-body induced coriolis-centripetal matrix
$C_A$	Added mass induced coriolis-centripetal matrix
$D( \nu )$	Damping matrix
$g(\eta)$	Restoring forces and moments vector
$\tau_{env}$	Environmental disturbances(wind,waves and currents)
$\tau_{hydro}$	Vector of hydrodynamic forces and moments
$\tau_{pro}$	Propeller forces and moments vector

## AUV Ciscrea model



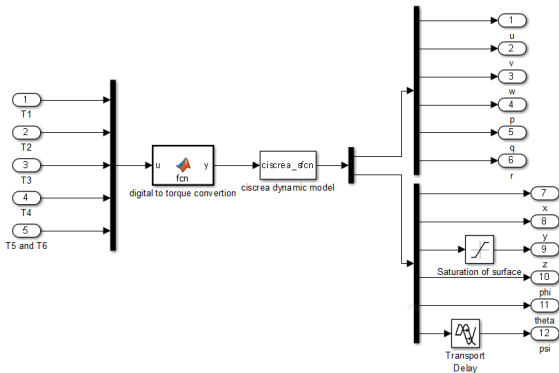
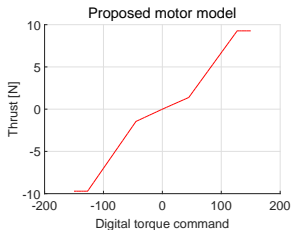
- 1 6 degrees of freedom.
- 2 4 degrees controllable.

## AUV Ciscrea model

$$\left\{ M_A + \begin{pmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_k \end{pmatrix} \right\} \begin{pmatrix} \ddot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n [T_i \cos(\psi_{t_i}) \cos(\theta_{t_i})] - (m - \rho v_{ol})g \sin(\theta) - D_{Nu}|u|u + D_{Lv}u + m(rv - qw) \\ \sum_{i=1}^n [T_i \sin(\psi_{t_i}) \cos(\theta_{t_i})] + (m - \rho v_{ol})g \cos(\theta) \sin(\phi) + D_{Nv}|v|v + D_{Lv}v + m(pw - ru) \\ - \sum_{i=1}^n [T_i \sin(\theta_{t_i})] + (m - \rho v_{ol})g \cos(\theta) \cos(\phi) - D_{Nw}|w|w + D_{Lw}w + m(qu - pv) \\ - \sum_{i=1}^n [T_i (y_{t_i} \sin(\theta_{t_i}) + z_{t_i} \sin(\psi_{t_i}) \cos(\theta_{t_i}))] - D_{Np}|p|p + D_{Lp}p + (I_y - I_z)qr \\ \sum_{i=1}^n [T_i (z_{t_i} \cos(\psi_{t_i}) \cos(\theta_{t_i}) + x_{t_i} \sin(\theta_{t_i}))] - D_{Nq}|q|q + D_{Lq}q + (I_z - I_x)rp \\ \sum_{i=1}^n [T_i (x_{t_i} \sin(\psi_{t_i}) \cos(\theta_{t_i}) - y_{t_i} \cos(\psi_{t_i}) \cos(\theta_{t_i}))] - D_{Nr}|r|r + D_{Lr}r + (I_x - I_y)pq \end{pmatrix} \quad (4)$$

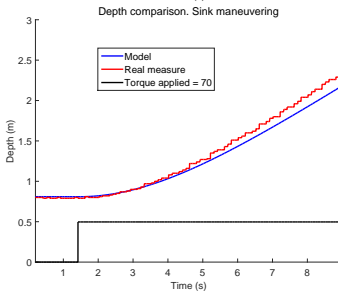
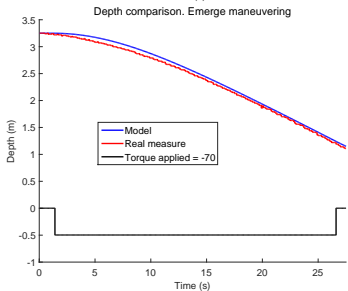
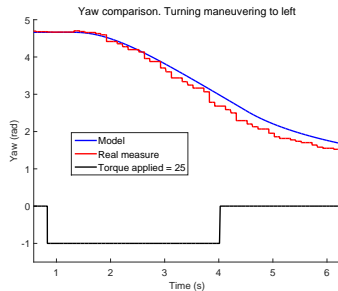
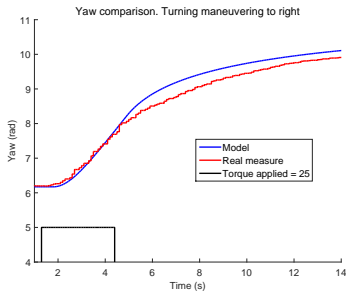
- ① Position of each propeller is considered.
- ② Cross relations between equations due to the angular momentum are considered.

# Simulator



- 1 Conversion of torque value.
- 2 Delay in the Yaw measure.

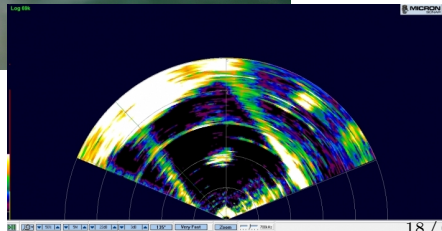
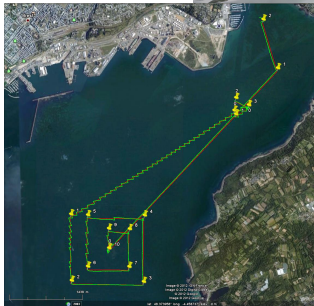
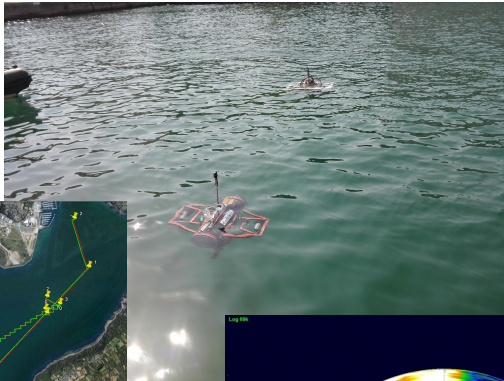
## Model validation



# Plan

- 1 Introduction.
- 2 AUV Ciscreea modeling.
- 3 Input constraint compensating algorithm.**
- 4 Output constraint compensating algorithm.
- 5 Control design under structural constraints.
- 6 Control design under dynamic constraints.
- 7 Concluding remarks.

# Case study: Cisrea AUV path following.



## Issues.

In most of the cases:

- ① Path to follow is given as vector input that can be parametrized in terms of a motion parameter.

The objective:

- ① Bounded path error.
- ② Minimal execution time.

The problems:

- ① Saturation of actuators.
- ② Regulation of the speed reference.

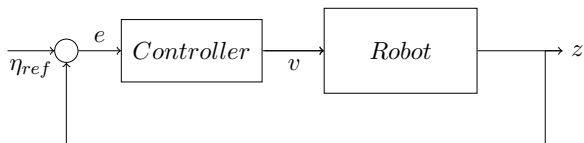


# Sliding mode motion parameter adaption (SMMPA).



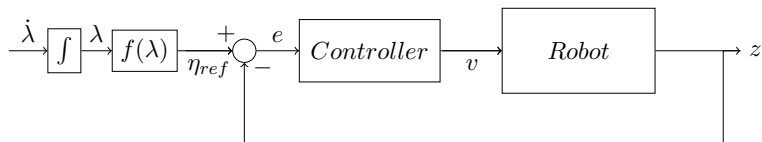
## 1 Modeling.

## Sliding mode motion parameter adaption (SMMPA).



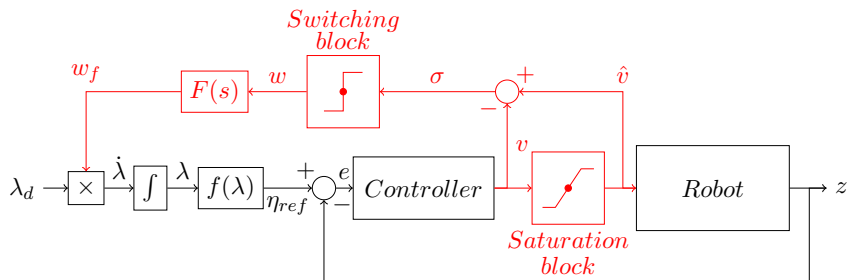
- 1 Modeling.
- 2 Classic feedback control.

## Sliding mode motion parameter adaption (SMMPA).



- 1 Modeling.
- 2 Classic feedback control.
- 3 Reference parametrization.

## Sliding mode motion parameter adaption (SMMPA).



- ①  $F(s)$  : First order low pass filter.
- ②  $\lambda_d$  : Reference speed parameter.
- ③  $f(\lambda)$  : Parametrization of the path.
- ④ *Controller* : PD controller.

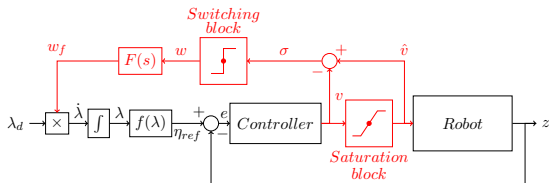
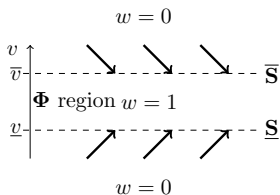
## Sliding mode motion parameter adaption (SMMPA).

### 1 Switching law:

$$w = \begin{cases} 1 & \text{if } \sigma = 0 \\ 0 & \text{if } \sigma \neq 0 \end{cases} \quad (5)$$

With:

$$\sigma(v) = v - \tilde{v} \quad (6)$$



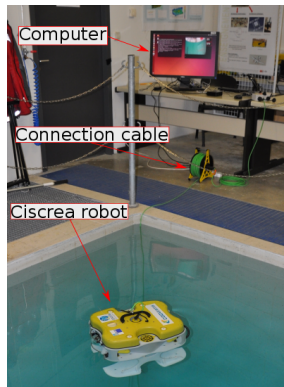
## Experimental setup.

Two situations are proposed:

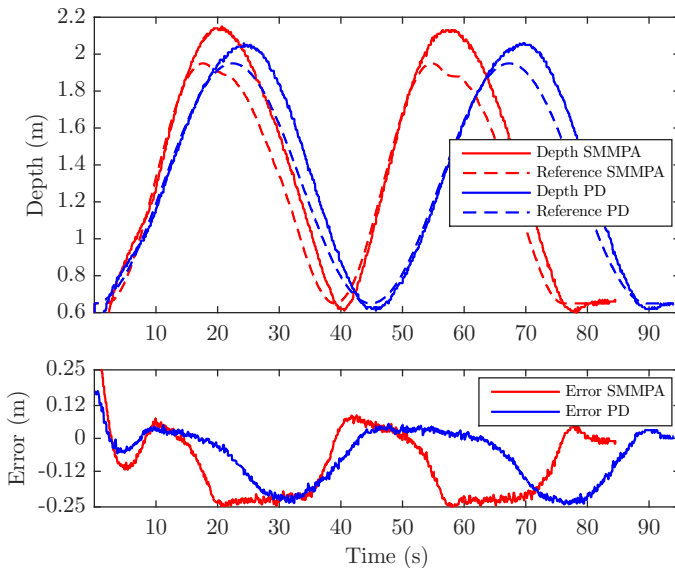
- 1 PD controller tuned to avoid actuator saturation.
- 2 The previous PD controller plus the proposed loop. Tuned so as to have the same order of path error.

Parameter values:

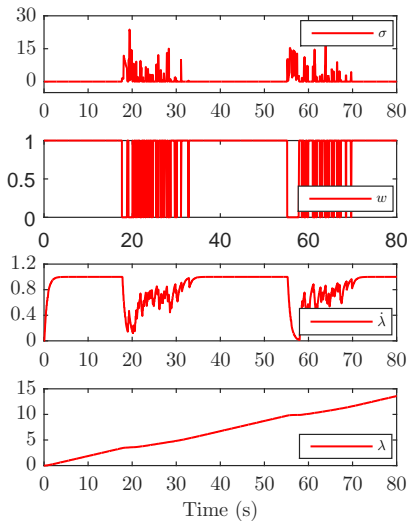
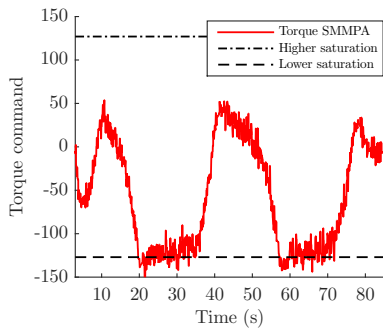
- 1  $\lambda_d = 0.2$
- 2 Cutoff frequency of the low pass filter  
 $f_c = 0.2387H_z$
- 3 PD controller:  
 $K_p = 541.43$ ,  $K_d = 250$ ,  
and  $f_f = 2.3H_z$
- 4 Step time: 0.1 seconds



## Experimental results - Path following comparison.



## Experimental results - Loop signals.





# Plan

- 1 Introduction.
- 2 AUV Ciscrea modeling.
- 3 Input constraint compensating algorithm.
- 4 Output constraint compensating algorithm.**
- 5 Control design under structural constraints.
- 6 Control design under dynamic constraints.
- 7 Concluding remarks.

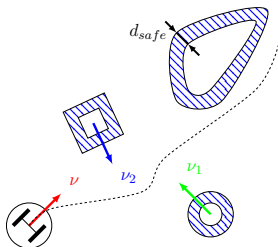
## Case study: strict path following.



### Objectives:

- 1 To follow a restricted path with minimum error in an unknown environment.
- 2 Minimal execution time.
- 3 Obstacle avoidance considering maximum dynamics that the vehicle must respect.

## Problem description

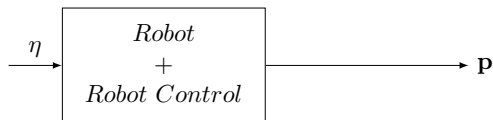


Given:

- ①  $\Psi$  subset of the environment that contains the obstacles.
- ②  $\hat{\Psi}$  extension of the subset  $\Psi$  that considers the safety margin  $d_{safe}$ .
- ③  $d(t)$  minimum distance from the robot to the subset  $\Psi$ .

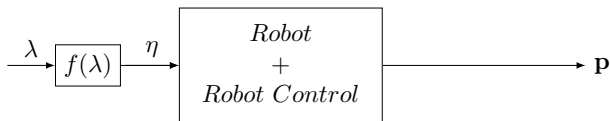
$$d(t) := \min_{\mathbf{r} \in \Psi} \|\mathbf{r} - \mathbf{p}(t)\| \quad (7)$$

## Collision avoidance speed adaption (CASA)



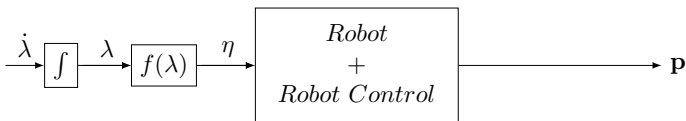
- 1 Robot.

## Collision avoidance speed adaption (CASA)



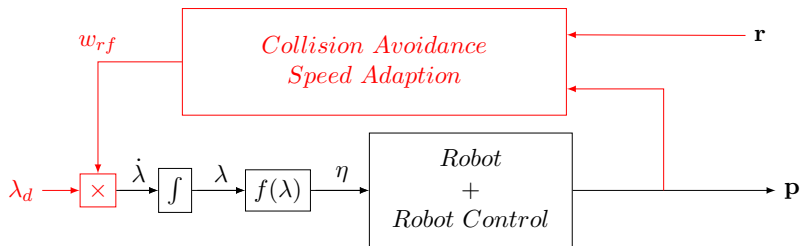
- 1 Robot.
- 2 Path parametrization.

## Collision avoidance speed adaption (CASA)



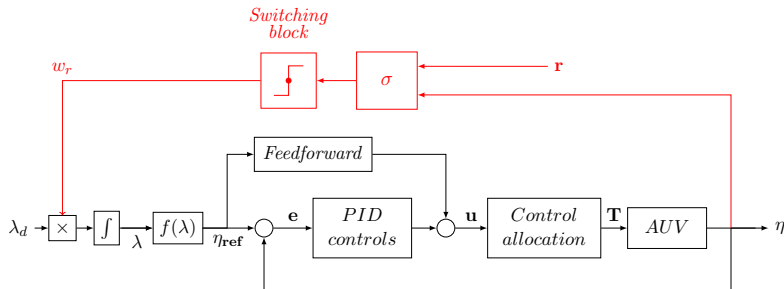
- 1 Robot.
- 2 Path parametrization.

## Collision avoidance speed adaption (CASA)



- ① Robot.
- ② Path parametrization.
- ③ Obstacle avoidance algorithm.

## Implementation in AUV Ciscarea.



- ① Minimal distance

$$d(t) := \min_{\mathbf{r} \in \Psi} \|\mathbf{r} - \eta(t)\|$$

- ② Sliding function

$$\sigma = d_{safe} - k_d d - k_{dd} \dot{d}$$

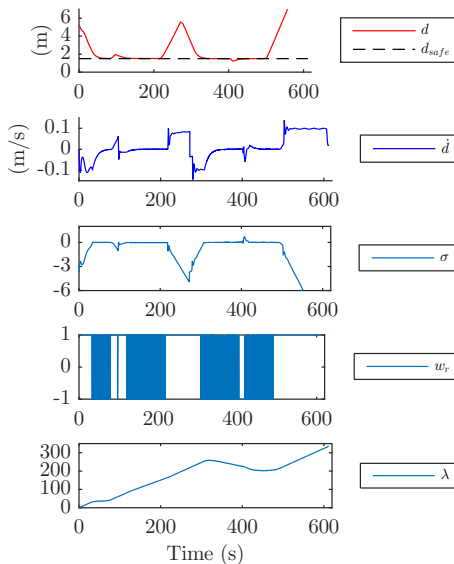
- ③ Switching function

$$w_r = \begin{cases} 1 & \sigma \leq 0 \\ -1 & \sigma > 0 \end{cases}$$



## Implementation in AUV Ciscree - simulations.

# Implementation in AUV Ciscrea - Loop signals.



## Plan

- 1 Introduction.
- 2 AUV Ciscreea modeling.
- 3 Input constraint compensating algorithm.
- 4 Output constraint compensating algorithm.
- 5 Control design under structural constraints.**
- 6 Control design under dynamic constraints.
- 7 Concluding remarks.

## Structural constraints.

- The performance of the previous auxiliary-loop techniques relies on the tune of a main controller.
- The controller is frequently predefined in industrial or commercial robots.

## Structural constraints.

- The performance of the previous auxiliary-loop techniques relies on the tune of a main controller.
- The controller is frequently predefined in industrial or commercial robots.

### Proposal:

- To tune a PID structured controller from  $H_\infty$  specification.
- A global optimization approach which enables performing a robustness analysis in a guaranteed way based on Interval Analysis.

## Robustness analysis.

- ① Let  $G(\sigma)$  be a LTI system which depends on real uncertain parameters  $\sigma \in \Sigma$ , where  $\Sigma$  denotes the set of admissible value of uncertainties.
- ② Suppose that a controller  $K$  was synthesized for a nominal plant  $G(\sigma_n)$  from constraints of the kind  $\mathcal{C}(G, K) \leq 0$ .  
Remembering:

- The stability constraint:  $R_i(\sigma) \leq 0$  using the Routh-Hurwitz criterion.
- The  $H_\infty$  constraints can be formulated as the modulus of a transfer function  $T$ ,  $|T(\sigma, i\omega)| - 1 \leq 0$ .

## Robustness analysis.

The proposed robustness analysis consists in verifying that the constraints are respected for all uncertainty values:

$$\text{Prove that } \mathcal{C}(G(\sigma), K) \leq 0, \forall \sigma \in \Sigma \quad (8)$$

---

<sup>1</sup>Monnet, D et al. (2016) A global optimization approach to structured regulation design under  $H_\infty$  constraints.

## Robustness analysis.

The proposed robustness analysis consists in verifying that the constraints are respected for all uncertainty values:

$$\text{Prove that } \mathcal{C}(G(\sigma), K) \leq 0, \forall \sigma \in \Sigma \quad (8)$$

As these constraints are not convex with structured controllers  $\Rightarrow$  a global optimization approach based on interval arithmetic is used<sup>1</sup>:

$$\sup_{\sigma \in \Sigma, \omega \in \Omega} \mathcal{C}(G(\sigma, i\omega), K(i\omega)) \quad (9)$$

where  $\Omega$  is a bounded interval of  $\mathbb{R}^+$

---

<sup>1</sup>Monnet, D et al. (2016) A global optimization approach to structured regulation design under  $H_\infty$  constraints.



## Case study: AUV Cisrea yaw direction control.

A linear system is needed:

- 1 Dismissing coupling effects between directions.
- 2 Linearizing the non-linear system, the non-linear behavior of actuators and the compass delay.

## Case study: AUV Cisrea yaw direction control.

A linear system is needed:

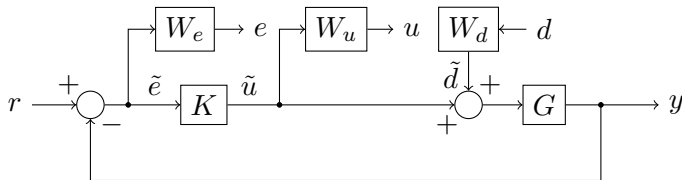
- 1 Dismissing coupling effects between directions.
- 2 Linearizing the non-linear system, the non-linear behavior of actuators and the compass delay.

The Yaw dynamic can be represented by the transfer function:

$$\frac{\psi(s)}{r(s)} = \frac{0.3931}{s^2 + 2.08\delta s} \frac{1 - 0.25s}{1 + 0.25s} \quad (10)$$

where  $\delta$  is the yaw angular velocity at which the system is linearized.

## Control design objectives.



These lead to the following synthesis problem, where if the norms are under 1, then the specifications are guaranteed.

Find  $K$  such as  $\alpha$  is minimum

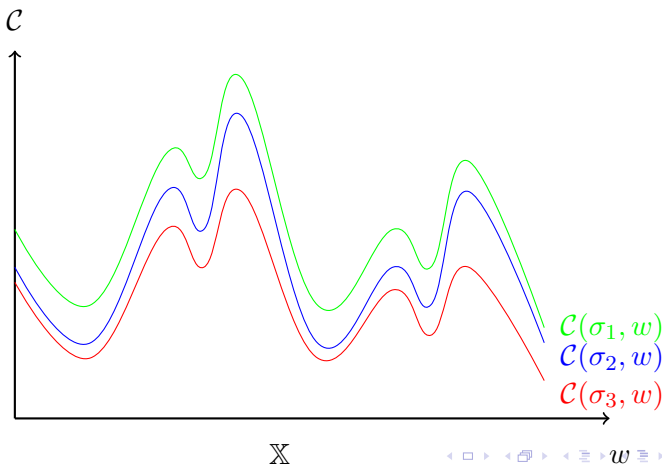
$$\begin{cases} \|W_e T_{r \rightarrow \tilde{e}}\|_{\infty} \leq \alpha, \\ \|W_e T_{\tilde{d} \rightarrow \tilde{e}} W_d\|_{\infty} \leq \alpha, \\ \|W_u T_{r \rightarrow \tilde{u}}\|_{\infty} \leq \alpha, \\ K \text{ stabilizes the closed-loop system.} \end{cases} \quad (11)$$

with

$$W_e(s) = \frac{0.1s+0.6283}{s+0.06283}, \quad W_d(s) = \frac{0.1s+0.06283}{s+0.6283}, \quad W_u = 0.167.$$

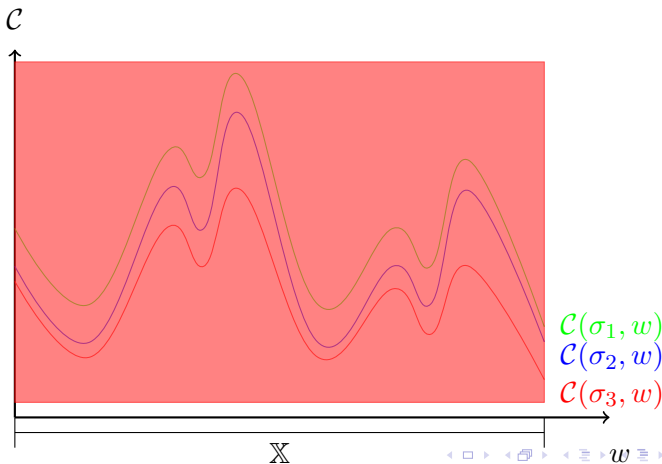
## Interval arithmetic &amp; interval branch and bound algorithm (IBBA).

$$\sup_{\sigma \in \Sigma, \omega \in \Omega} \mathcal{C}(G(\sigma, i\omega), K(i\omega))$$



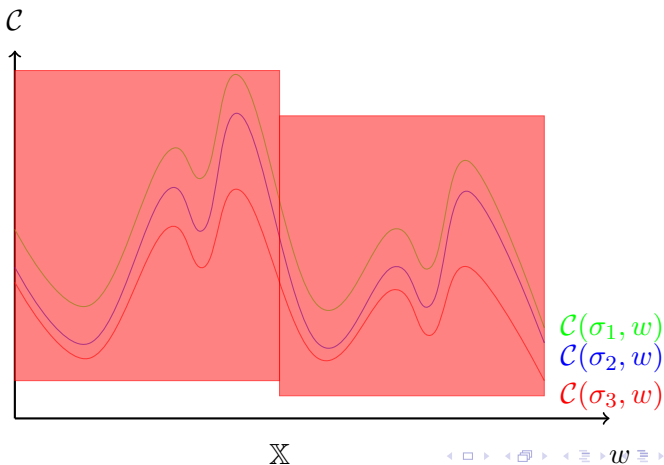
## Interval arithmetic &amp; interval branch and bound algorithm (IBBA).

$$\sup_{\sigma \in \Sigma, \omega \in \Omega} \mathcal{C}(G(\sigma, i\omega), K(i\omega))$$



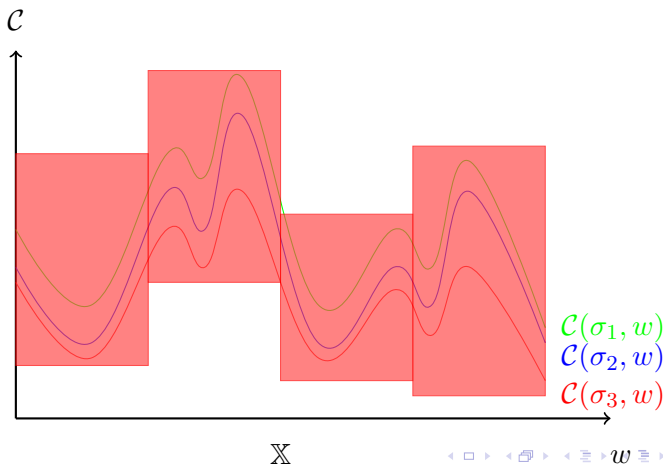
## Interval arithmetic &amp; interval branch and bound algorithm (IBBA).

$$\sup_{\sigma \in \Sigma, \omega \in \Omega} \mathcal{C}(G(\sigma, i\omega), K(i\omega))$$



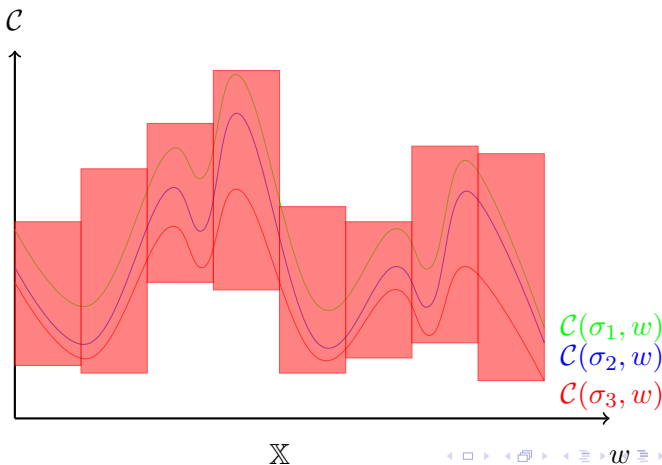
## Interval arithmetic &amp; interval branch and bound algorithm (IBBA).

$$\sup_{\sigma \in \Sigma, \omega \in \Omega} \mathcal{C}(G(\sigma, i\omega), K(i\omega))$$



## Interval arithmetic &amp; interval branch and bound algorithm (IBBA).

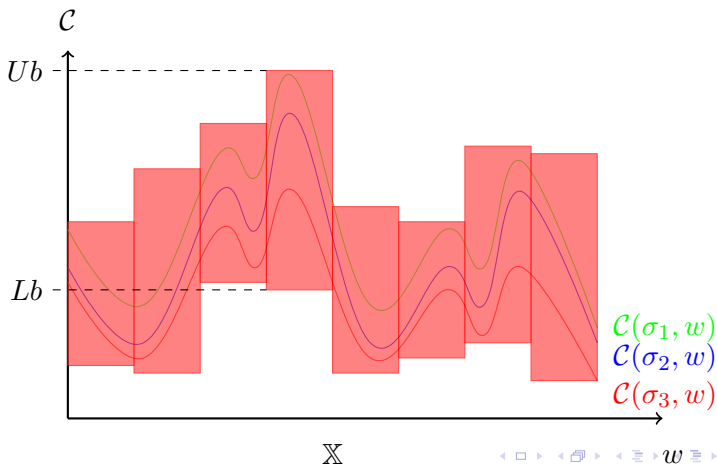
$$\sup_{\sigma \in \Sigma, \omega \in \Omega} \mathcal{C}(G(\sigma, i\omega), K(i\omega))$$





## Interval arithmetic &amp; interval branch and bound algorithm (IBBA).

$$\sup_{\sigma \in \Sigma, \omega \in \Omega} \mathcal{C}(G(\sigma, i\omega), K(i\omega))$$



## Control design.

- We propose to synthesize a PID controller with a particular plant  $G(\tilde{\delta})$ , with  $\delta = \tilde{\delta} = 2$ .
- The PID controller has the form:  $K(k, s) = k_p + \frac{k_i}{s} + \frac{k_d s}{1+Ts}$  with  $k = (k_p, k_i, k_d, T)$ .
- The Matlab's toolbox Systune provides the following solution:

$$\tilde{k} = (4.68, 0.71, 4.68, 0.11).$$

- The control law is robust if both stability and  $H_\infty$  constraints are respected for all  $\delta \in [0, 4]$ .

## Analysis results.

The stability of the closed-loop system can be expressed as a set of four polynomial inequalities with the Routh-Hurwitz criterion:

$$\sup_{\delta \in [0,4]} R_i(\delta, \tilde{k}) \leq -0.01, \forall i \in \{1, \dots, 4\},$$

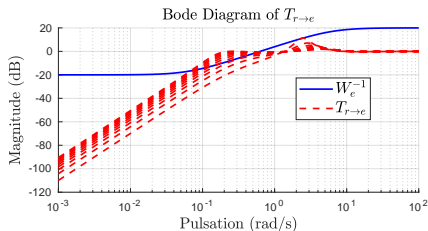
This proves that  $K(\tilde{k})$  robustly stabilizes the linear closed-loop system.

The robustness analysis of  $H_\infty$  constraints over the pulsation range  $[0, \omega_c]$  provides the following results:

$$\sup_{\delta \in [0,4]} \{ \|W_e T_{r \rightarrow \tilde{e}}(\tilde{k})\|_\infty \} \in [6.55, 7.20]$$

$$\sup_{\delta \in [0,4]} \{ \|W_e T_{d \rightarrow \tilde{e}}(\tilde{k}) W_d\|_\infty \} \leq 0.56$$

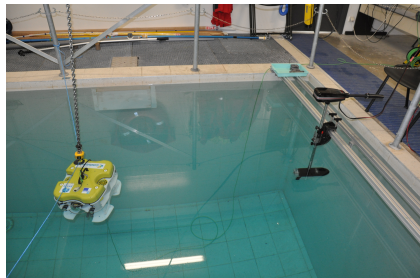
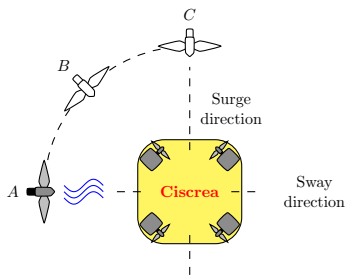
$$\sup_{\delta \in [0,4]} \{ \|W_u T_{r \rightarrow \tilde{u}}(\tilde{k})\|_\infty \} \leq 0.89$$



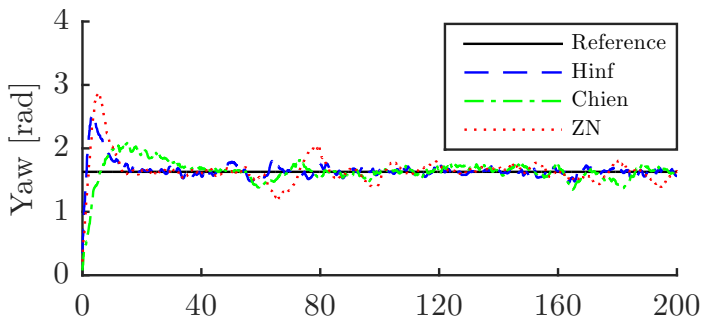
## Experiment setup

Another two classical controllers are compared over simulations and real experiments to the one proposed here.

- *ZN* controller :  $k_{ZN} = (1.32, 0.22, 1.89, 0.5)$ .
- *Chien* controller:  $k_{Chien} = (1.82, 0.12, 6.4, 0.35)$ .

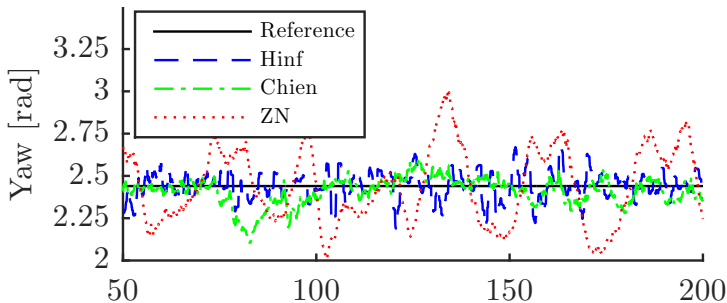


## Experimental results - Perturbation in sway direction



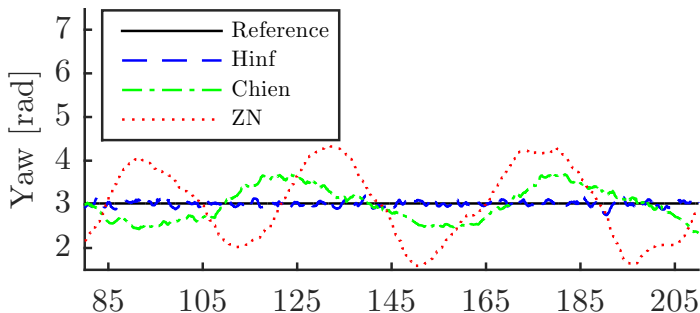
Experiment	RMSE	NMAE	BIAS
ZN	0.2166	0.0689	-0.0204
<b>Hinf</b>	<b>0.1355</b>	<b>0.0386</b>	<b>-0.0230</b>
Chien	0.1738	0.0762	-0.0137

## Experimental results - Perturbation at 45 degrees of surge direction



Experiment	RMSE	NMAE	Bias
ZN	0.1742	0.0502	0.0137
<b>Hinf</b>	<b>0.0650</b>	<b>0.0174</b>	<b>0.0037</b>
Chien	0.0755	0.0179	0.0172

## Experimental results - Perturbation in surge direction



Experiment	RMSE	NMAE	Bias
ZN	0.3957	0.0749	-0.0037
<b>Hinf</b>	<b>0.0371</b>	<b>0.0059</b>	<b>-7.1612e-04</b>
Chien	0.2548	0.0482	0.0256

## Plan

- 1 Introduction.
- 2 AUV Ciscreea modeling.
- 3 Input constraint compensating algorithm.
- 4 Output constraint compensating algorithm.
- 5 Control design under structural constraints.
- 6 Control design under dynamic constraints.**
- 7 Concluding remarks.



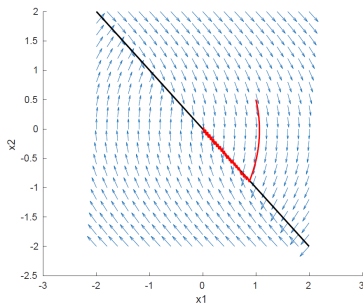
## Dynamic constraints.

Idea:

- To use global optimization and interval arithmetic to synthesize SM robust controllers for nonlinear systems.
- To have a measure of this robustness.

Looking back on sufficient conditions for SM establishment:

$$\begin{cases} \dot{\sigma}(\mathbf{x}) < \mathbf{0} & \text{if } \sigma(\mathbf{x}) > \mathbf{0} \\ \dot{\sigma}(\mathbf{x}) > \mathbf{0} & \text{if } \sigma(\mathbf{x}) < \mathbf{0} \end{cases}$$



## Sliding mode problems.

Equivalent control over the sliding surface  $\sigma = 0$ :

$$u_{eq}(x) = -\frac{L_f \sigma}{L_g \sigma}$$

The sliding condition holds if:

$$u^- \leq u_{eq}(x) \leq u^+$$

### Analysis problem

Do the sliding sufficient conditions hold over  $\mathbb{X} \subseteq \mathbb{R}^{n_x}$ ?

### Synthesis problem

Given  $u^+$  and  $u^-$ , what is the fastest dynamic that can be achieved?

Both imply to compute the minimum and the maximum of  $u_{eq}$  over  $\mathbb{X}$ .  $\rightarrow$  But  $u_{eq}$  is non-convex in the general case.

$\rightarrow$  Global optimization tools are needed.

## Global optimization problem formulation.

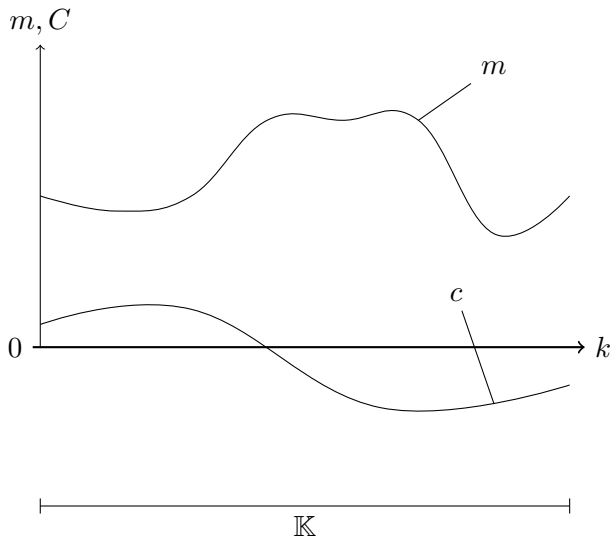
It is possible to formulate:

$$\begin{cases} \inf_{\mathbf{k} \in \mathbb{R}^n} & m(\mathbf{k}) \\ \text{subject to} & c(\mathbf{k}) \leq 0, \end{cases} \quad (12)$$

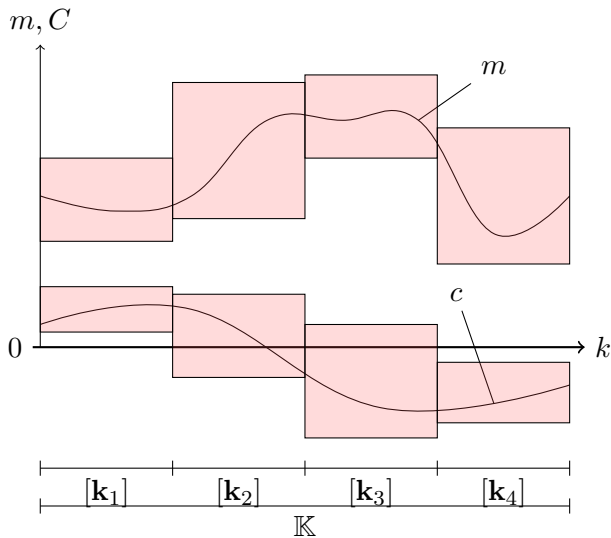
here:

- $m (\mathbb{R}^n \rightarrow \mathbb{R})$  is the objective function.
- $\mathbf{k} \in \mathbb{R}^n$  is the optimization variable.
- $c (\mathbb{R}^n \rightarrow \mathbb{R})$  is a function which defines a subset where the solution is searched.

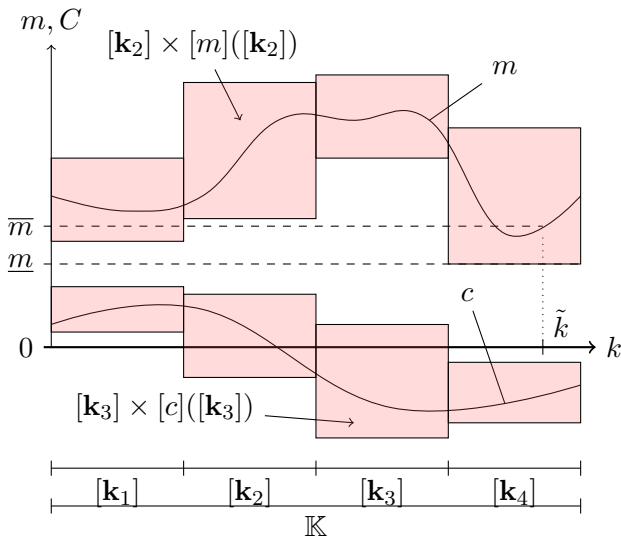
# GO, IBBA and set inversion via interval analysis (SIVIA).



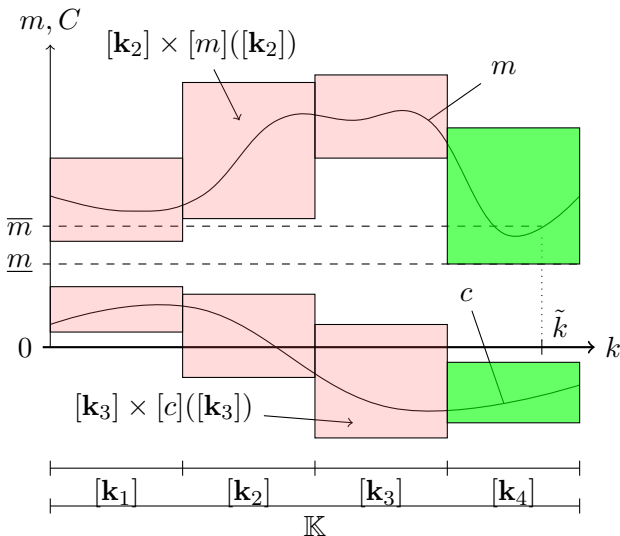
## GO, IBBA and set inversion via interval analysis (SIVIA).



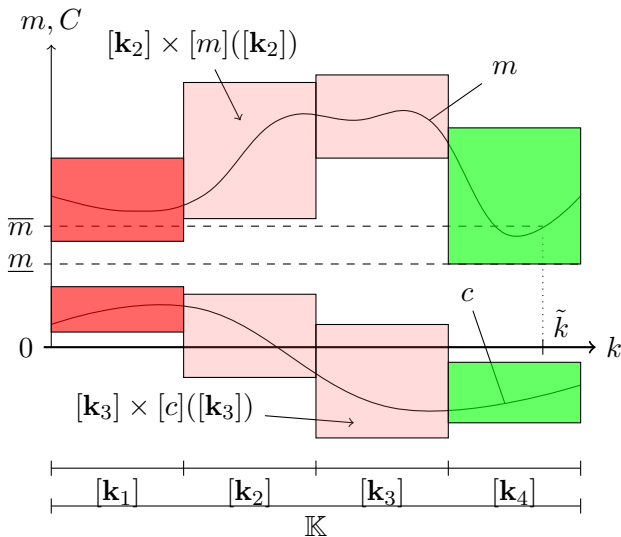
## GO, IBBA and set inversion via interval analysis (SIVIA).



## GO, IBBA and set inversion via interval analysis (SIVIA).

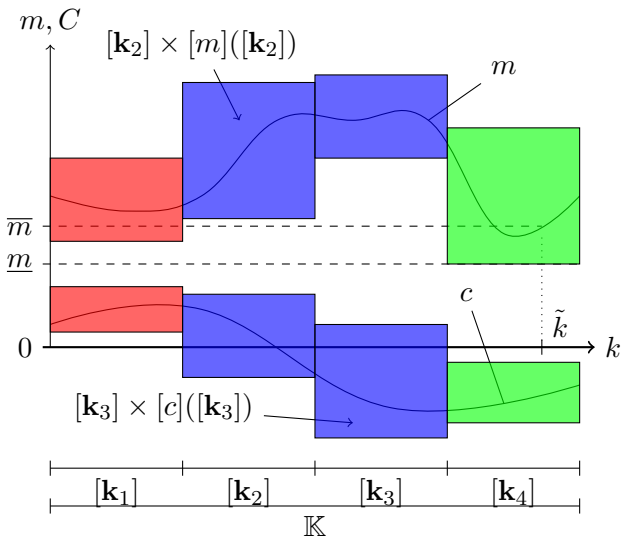


## GO, IBBA and set inversion via interval analysis (SIVIA).





## GO, IBBA and set inversion via interval analysis (SIVIA).



## Case study: AUV Cisrea heave control.

Formulating a SM control with a desired dynamics of the form:

$$\sigma = \dot{e} + \lambda e = 0 \quad (13)$$

with:

- $e = z_d - z$  the tracking error.
- $z_d$  the reference position.
- $\lambda$  an approaching rate tuning parameter.

Implemented with the discontinuous control action:

$$\tau_{pro} = |\tau_{max}| \text{sign}(\sigma) \quad \text{with :} \quad |u^-| = |u^+| = \tau_{max} = 6 \text{ Nm} \quad (14)$$

The equivalent control signal is:

$$u_{eq} = -\lambda \dot{z} (M_{RB} + M_A) + D_L \dot{z} + D_{NL} |\dot{z}| \dot{z} + g(z) - \tau_{env} \quad (15)$$

## Synthesis problem.

### Synthesis problem

Given  $u^+$  and  $u^-$ , what is the fastest dynamic that can be achieved?

## Synthesis problem.

### Synthesis problem

Given  $u^+$  and  $u^-$ , what is the fastest dynamic that can be achieved?

Program to solve:

$$\left\{ \begin{array}{l} \sup_{\lambda \in \Delta} \lambda \\ \text{s. t.} \quad u^- \leq u_{eq}(\lambda, x), \forall x \in \mathbb{X} \\ \quad \quad u_{eq}(\lambda, x) \leq u^+, \forall x \in \mathbb{X} \end{array} \right.$$

→ Semi infinite program (SIP).

Or equivalently

$$\left\{ \begin{array}{l} \sup_{\lambda \in \Delta} \lambda \\ \text{s. t.} \quad u^- \leq \min_{x \in \mathbb{X}} u_{eq}(\lambda, x), \\ \quad \quad \sup_{x \in \mathbb{X}} u_{eq}(\lambda, x) \leq u^+. \end{array} \right.$$

## Synthesis problem.

### Synthesis problem

Given  $u^+$  and  $u^-$ , what is the fastest dynamic that can be achieved?

$$\left\{ \begin{array}{l} \sup_{\lambda \in \Delta} \lambda \\ \text{s. t.} \quad u^- \leq u_{eq}(\lambda, \dot{z}, \tau_{env}), \forall \dot{z} \in [\dot{z}], \forall \tau_{env} \in [\tau_{env}] \\ \quad \quad u_{eq}(\lambda, \dot{z}, \tau_{env}) \leq u^+, \forall \dot{z} \in [\dot{z}], \forall \tau_{env} \in [\tau_{env}] \end{array} \right.$$

With:

- $[\dot{z}] = [-0.15, 0.15]$
- $[\tau_{env}] = [-3, 3]$

Global maximum over  $\Delta = [0, 2]$ :

## Synthesis problem.

### Synthesis problem

Given  $u^+$  and  $u^-$ , what is the fastest dynamic that can be achieved?

$$\left\{ \begin{array}{l} \sup_{\lambda \in \Delta} \lambda \\ \text{s. t.} \quad u^- \leq u_{eq}(\lambda, \dot{z}, \tau_{env}), \forall \dot{z} \in [\dot{z}], \forall \tau_{env} \in [\tau_{env}] \\ \quad \quad u_{eq}(\lambda, \dot{z}, \tau_{env}) \leq u^+, \forall \dot{z} \in [\dot{z}], \forall \tau_{env} \in [\tau_{env}] \end{array} \right.$$

With:

- $[\dot{z}] = [-0.15, 0.15]$
- $[\tau_{env}] = [-3, 3]$

Global maximum over  $\Delta = [0, 2]$ :

$$\lambda \in [0.3842, 0.3885]$$

## Analysis problem I.

### Analysis problem

Given  $u^+, u^-$  and  $\lambda \in \Delta$ , what are the speeds at which SM can be achieved?

## Analysis problem I.

### Analysis problem

Given  $u^+, u^-$  and  $\lambda \in \Delta$ , what are the speeds at which SM can be achieved?

Characterize the feasible set of CSP

$$\begin{cases} u^- \leq u_{eq}(\lambda, \dot{z}, \tau_{env}), \forall \tau_{env} \in [\tau_{env}] \\ u_{eq}(\lambda, \dot{z}, \tau_{env}) \leq u^+, \forall \tau_{env} \in [\tau_{env}] \end{cases}$$

With:

- $|u^-| = |u^+| = \tau_{max} = 6 \text{ Nm}$
- $[\tau_{env}] = [-3, 3]$

Search domain:

- $\Delta = [0, 2]$
- $[\dot{z}] = [-1, 1]$



## Analysis problem I.

### Analysis problem

Given  $u^+, u^-$  and  $\lambda \in \Delta$ , what are the speeds at which SM can be achieved?

Characterize the feasible set of CSP

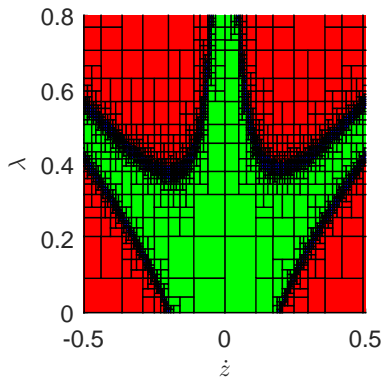
$$\begin{cases} u^- \leq u_{eq}(\lambda, \dot{z}, \tau_{env}), \forall \tau_{env} \in [\tau_{env}] \\ u_{eq}(\lambda, \dot{z}, \tau_{env}) \leq u^+, \forall \tau_{env} \in [\tau_{env}] \end{cases}$$

With:

- $|u^-| = |u^+| = \tau_{max} = 6 \text{ Nm}$
- $[\tau_{env}] = [-3, 3]$

Search domain:

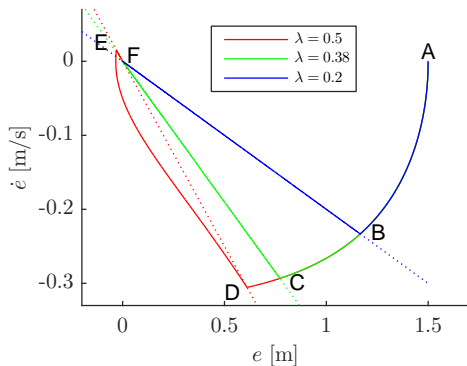
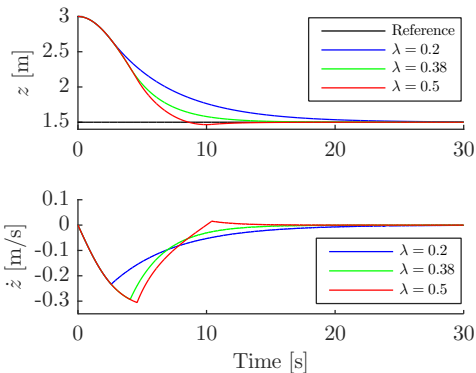
- $\Delta = [0, 2]$
- $[\dot{z}] = [-1, 1]$



## Analysis problem I - System step response.

### Analysis problem

Given  $u^+, u^-$  and  $\lambda \in \Delta$ , what are the speeds at which SM can be achieved?



## Analysis problem II.

### Analysis problem

What happens if we do not know exactly a parameter of the system?

Characterize the feasible set of CSP

$$\begin{cases} u^- \leq u_{eq}(\lambda, \dot{z}, \tau_{env}, M_A), & \forall \tau_{env} \in [\tau_{env}], \forall M_A \in [M_A] \\ u_{eq}(\lambda, \dot{z}, \tau_{env}, M_A) \leq u^+, & \forall \tau_{env} \in [\tau_{env}], \forall M_A \in [M_A] \end{cases}$$

## Analysis problem II.

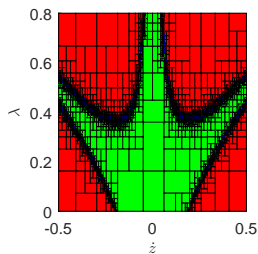
### Analysis problem

What happens if we do not know exactly a parameter of the system?

Characterize the feasible set of CSP

$$\begin{cases} u^- \leq u_{eq}(\lambda, \dot{z}, \tau_{env}, M_A), & \forall \tau_{env} \in [\tau_{env}], \forall M_A \in [M_A] \\ u_{eq}(\lambda, \dot{z}, \tau_{env}, M_A) \leq u^+, & \forall \tau_{env} \in [\tau_{env}], \forall M_A \in [M_A] \end{cases}$$

5%  $M_A$  variation.



## Analysis problem II.

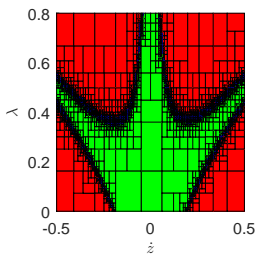
### Analysis problem

What happens if we do not know exactly a parameter of the system?

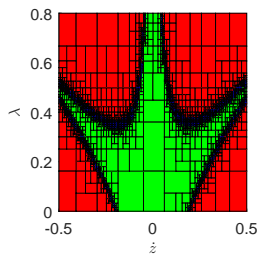
Characterize the feasible set of CSP

$$\begin{cases} u^- \leq u_{eq}(\lambda, \dot{z}, \tau_{env}, M_A), & \forall \tau_{env} \in [\tau_{env}], \forall M_A \in [M_A] \\ u_{eq}(\lambda, \dot{z}, \tau_{env}, M_A) \leq u^+, & \forall \tau_{env} \in [\tau_{env}], \forall M_A \in [M_A] \end{cases}$$

5%  $M_A$  variation.



10%  $M_A$  variation.



## Analysis problem II.

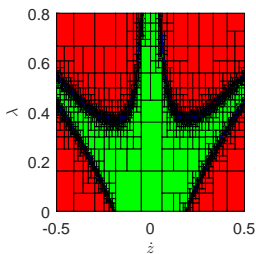
### Analysis problem

What happens if we do not know exactly a parameter of the system?

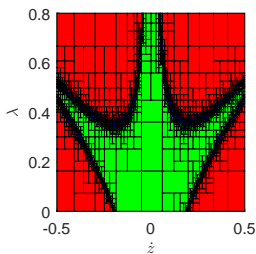
Characterize the feasible set of CSP

$$\begin{cases} u^- \leq u_{eq}(\lambda, \dot{z}, \tau_{env}, M_A), & \forall \tau_{env} \in [\tau_{env}], \forall M_A \in [M_A] \\ u_{eq}(\lambda, \dot{z}, \tau_{env}, M_A) \leq u^+, & \forall \tau_{env} \in [\tau_{env}], \forall M_A \in [M_A] \end{cases}$$

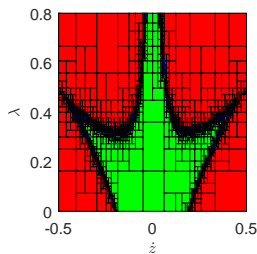
5%  $M_A$  variation.



10%  $M_A$  variation.



25%  $M_A$  variation.



## Analysis problem III.

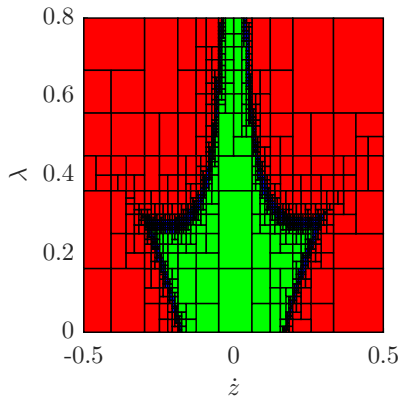
### Analysis problem

What happens if we do not know exactly several parameters of the system? (25% variation on  $M_A$  and  $D_{NL}$  nominal values)

## Analysis problem III.

## Analysis problem

What happens if we do not know exactly several parameters of the system? (25% variation on  $M_A$  and  $D_{NL}$  nominal values)





## Analysis problem IV.

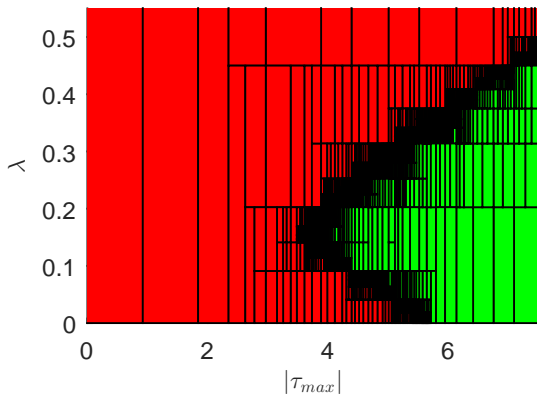
### Analysis problem

Given  $\tau_{max} \in [\tau_{max}]$  and  $\lambda \in \Delta$ , what is the minimal control action we need for a desired  $\lambda$  value?

## Analysis problem IV.

## Analysis problem

Given  $\tau_{max} \in [\tau_{max}]$  and  $\lambda \in \Delta$ , what is the minimal control action we need for a desired  $\lambda$  value?



## Plan

- 1 Introduction.
- 2 AUV Ciscreea modeling.
- 3 Input constraint compensating algorithm.
- 4 Output constraint compensating algorithm.
- 5 Control design under structural constraints.
- 6 Control design under dynamic constraints.
- 7 Concluding remarks.

## Particular conclusions - results

- Development of mitigation techniques for input/output constraints, adaptable to already implemented systems.
- Development of a robust tuning technique for PID structure controllers.
- Combination of interval techniques and global optimization for the analysis of robustness of non-linear systems and, in particular for the tuning of SM designs.  
→ Formalization of the design problem of SM controllers as an SIP.

## Publications

- J.L. Rosendo, B. Clement, F. Garelli. Experimental validation of constraint mitigation algorithm in underwater robot depth control. Part I: Journal of Systems and Control Engineering. 2018 (DOI: 10.1177/0959651818791399).
- J.L. Rosendo, D. Monnet, B. Clement, F. Garelli, J. Ninin. Control of an Autonomous Underwater Vehicle subject to robustness constraints. 9th IFAC Symposium on Robust Control Design (ROCOND'18). Florianopolis, Brazil. IFAC-PapersOnLine: Volume 51, Issue 25, 2018, Pages 322-327.
- D. Monnet, J.L. Rosendo, H. De Battista, B. Clement, J. Ninin, F. Garelli. A global optimization approach for non-linear sliding mode control analysis and design. 9th IFAC Symposium on Robust Control Design (ROCOND'18). Florianopolis, Brazil. IFAC-PapersOnLine: Volume 51, Issue 25, 2018, Pages 128-133.
- J.L. Rosendo, D. Monnet, H. De Battista, J. Ninin, B. Clement, F. Garelli. Sliding mode control analysis and design for an AUV application using global optimization techniques. Nonlinear Dynamics (submitted).
- J.L. Rosendo, F. Garelli, H. De Battista. Obstacle avoidance with path restrictions in autonomous underwater vehicles. AADECA 2018 - Semana del Control Automático - 26° Congreso Argentino de Control Automático. Buenos Aires, Argentina.
- J.L. Rosendo, F. Garelli, H. De Battista, F. Valenciaga. Obstacle avoidance under strict path following. XVII Workshop on Information Processing and Control (RPIC). Argentina, Mar del Plata. 2017. DOI: 10.23919/RPIC.2017.8214344.
- J.L. Rosendo, B. Clement, F. Garelli. Acondicionamiento de la referencia utilizando modos deslizantes en aplicaciones de seguimiento de camino en AUV. Cuartas Jornadas de Investigación, Transferencia y Extensión de la Facultad de Ingeniería. UNLP. Facultad de Ingeniería, Universidad Nacional de La Plata. Argentina. La Plata.
- J.L. Rosendo, B. Clement, F. Garelli. Sliding mode reference conditioning for path following applied to an AUV. CAMS 2016. Norway. Trondheim. 2016. IFAC-PapersOnLine: Volume 49, Issue 23, 2016, Pages 8-13.
- J.L. Rosendo, D. Monnet, B. Clement, F. Garelli. Control of an Autonomous Underwater Vehicule under robustness constraints. Workshop. SWIM 2016 (Summer Workshop on Interval Methods). École Normale Supérieure de Lyon (ENS de Lyon). Francia. Lyon.
- J.L. Rosendo, F. Garelli, B. Clement, H. De Battista. Mitigation of the saturation effect in AUV path following applications. AADECA 2016. Argentina. Buenos Aires. 2016. ISBN 978-950-99994-9-7.

## Global conclusions - Acknowledgments

- An incursion in the robotics world was made from the point of view of control theory.
- Theoretical and practical knowledge was obtained that will serve as the basis for further projects in the area of robotics at UNLP.
- Collaboration was achieved with another research group, which is expected to continue in time.

## Global conclusions - Acknowledgments

- An incursion in the robotics world was made from the point of view of control theory.
- Theoretical and practical knowledge was obtained that will serve as the basis for further projects in the area of robotics at UNLP.
- Collaboration was achieved with another research group, which is expected to continue in time.

An especial thanks to:

- CONICET.
- UNLP - FI - LEICI.
- ENSTA Bretagne.
- Eiffel scholarship.

